

Intrinsic Cutoff and Acausality for Massive Spin 2 Fields Coupled to Electromagnetism

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Abstract

We couple a massive spin 2 particle to electromagnetism. By introducing new, redundant degrees of freedom using the Stückelberg formalism, we extract an intrinsic, model independent UV cutoff of the effective field theory describing this system. The cutoff signals both the onset of a strongly interacting dynamical regime and a finite size for the spin 2 particle. We show that the existence of a cutoff is strictly connected to other pathologies of interacting high-spin fields, such as the Velo-Zwanziger acausality. We also briefly comment on implications of this result for the detection of high spin states and on its possible generalization to arbitrary spin.

1 Introduction

While the coupling of massless high-spin particles to electromagnetism or gravity is notoriously fraught with inconsistencies (see e.g. [1]), massive charged particles of any spin can and do exist. Massive higher-spin particles like $\pi_2(1670)$, $\rho_3(1690)$ or $a_4(2040)$ have amply been produced in particle colliders. They are resonances, thus composite and unstable. So they can be described by an effective, local field theory only up to some finite UV cutoff Λ , of the order of their inverse size. In known resonances, this cutoff is also of the same order of magnitude as their mass m .

String theory also predicts massive higher-spin particles, with mass at least as large as $\mathcal{O}(M_{string})$. These particles couple to $U(1)$ gauge fields or gravity, and can be given an effective field theory description, but again with a finite cutoff $\Lambda = \mathcal{O}(M_{string})$.

Both in string theory and in QCD, high spin states always interact with other states of lower spin and mass $\mathcal{O}(\Lambda)$. We may wonder if the approximate equality $\Lambda \approx m$ is just a property of these two examples, or if it is a general feature of charged high-spin particles. An answer to this question is relevant to figuring out possible experimental signatures of high-spin particles in future colliders. For instance, it rules out long-lived high-spin charged particles, and thus affects directly the strategy for their search.

In this paper we will begin a study of interacting high-spin massive particles, starting with a relatively simple yet interesting case: a spin two particle coupled to electromagnetism. The first task in constructing an effective field theory for a high-spin field is to write a free Lagrangian. This is known for massive fields of arbitrary spin. The earliest such Lagrangian was written a long time ago by Singh and Hagen [2]. Auxiliary fields are necessary to the best of our knowledge, unless the Lagrangian is nonlocal [3]. Gauge-invariant Lagrangians for free *massless* high-spin fields are also well-known [4]. Inconsistencies arise when one tries to make these fields interact. These inconsistencies are due to the absence of currents invariant under the high-spin gauge symmetry, as first clearly illustrated in the case of spin 5/2 coupled to gravity by Aragone and Deser in [1]¹.

Massive high-spin fields *can* be coupled to electromagnetism. After all, charged high-spin resonances do exist! This fact alone implies that any Lagrangian describing charged high-spin fields interacting with electromagnetism must be singular in the massless limit $m \rightarrow 0$. If one takes the Singh-Hagen Lagrangians and follows the minimal-coupling prescription to introduce electromagnetic interactions, then the massless singularity is far from manifest. Indeed, the resulting Lagrangians contain only positive powers of the mass. Scattering amplitudes become singular because in the massless limit the high-spin free Lagrangian acquires a gauge invariance that makes its kinetic term non-invertible. Correspondingly, the propagator of the massive theory also becomes singular when $m \rightarrow 0$. The problem is thus associated with the gauge invariance of the free theory. It thus manifests already for massive spin one; but in that case it can be cured by adding a non-minimal dipole term. For higher spins, some but not all singular terms can also be eliminated by adding non-minimal terms [5].

In this paper, we will argue that the massless singularity is indeed robust and cannot be completely canceled by adding non-minimal terms. We will also quantify the degree

¹The absence of gauge invariant currents also makes the coupling of massless $s = 3/2, 2$ fields to electromagnetism inconsistent.

of singularity of the massless limit. Specifically, for the case of spin 2, we will argue that the cutoff of the effective action is always lower than

$$\Lambda_2 \equiv \frac{m}{\sqrt{e}}, \quad (1)$$

where e is the electric charge. We will always work in flat space; equivalently, we will consider particles with mass much higher than the inverse curvature radius of the space-time background. For different values of the cosmological constant additional consistency bounds also apply [6].

A systematic study of the mass singularity is greatly facilitated by using the Stückelberg formalism, i.e., by making the massive free theory gauge invariant through the addition of auxiliary fields. These fields can be set to vanish using the resulting gauge invariance. In this case the Stückelberg action reduces to the original one. On the other hand, a different, judicious choice of (covariant) gauge fixing can make all kinetic terms in the theory canonical (diagonal on momentum eigenstates and proportional to $p^\mu p_\mu$). In this case, inverse powers of the mass appear explicitly in the (non-renormalizable) interaction terms involving the auxiliary fields.

A simple, standard example of the procedure is a complex, massive spin 1 field W_μ coupled to electromagnetism. The free Lagrangian is

$$L = -\frac{1}{2}G_{\mu\nu}^*G^{\mu\nu} - m^2W_\mu^*W^\mu, \quad G_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu. \quad (2)$$

The action becomes gauge invariant after a complex compensator scalar field ϕ is introduced by the substitution $W_\mu = V_\mu - \partial_\mu \phi/m$. After adding the gauge fixing term $-|\partial_\mu V^\mu - m\phi|^2$ the action eq. (2) becomes diagonal

$$L - |\partial_\mu V^\mu - m\phi|^2 = V_\mu^*(\square - m^2)V^\mu + \phi^*(\square - m^2)\phi. \quad (3)$$

The minimal substitution $\partial_\mu \rightarrow D_\mu \equiv \partial_\mu \pm ieA_\mu$ generates non-renormalizable interaction terms²:

$$\left[-i\frac{e}{2m}F_{\mu\nu}\phi^*(D^\mu V^\nu - D^\nu V^\mu) + c.c. \right] - \frac{e^2}{2m^2}F_{\mu\nu}F^{\mu\nu}\phi^*\phi. \quad (4)$$

Notice that these terms arise only from the kinetic term of Lagrangian (2), *not* from the gauge fixing.

The UV cutoff signaling the breakdown of our effective field theory is now explicit: it is the coupling constant multiplying the non-renormalizable interactions terms. To make this even clearer, we take the decoupling limit

$$m \rightarrow 0, \quad e \rightarrow 0, \quad \frac{m}{e} = \text{constant} \equiv \Lambda. \quad (5)$$

The Lagrangian does not become free, but instead reduces to

$$L = V_\mu^*\square V^\mu + \phi^*\square\phi + \left[-\frac{i}{2\Lambda}F_{\mu\nu}\phi^*(\partial^\mu V^\nu - \partial^\nu V^\mu) + c.c. \right] - \frac{1}{2\Lambda^2}F_{\mu\nu}F^{\mu\nu}\phi^*\phi. \quad (6)$$

²They are non-renormalizable because the fields V_μ, ϕ have canonical dimension one, and their kinetic terms are nonsingular and canonically normalized.

For spin one the cutoff Λ is not an intrinsic property of the theory, since all non-renormalizable terms are canceled by adding to the Lagrangian a non-minimal (dipole) term

$$L_{NM} = ieF_{\mu\nu}W^{*\mu}W^\nu. \quad (7)$$

This term is power counting renormalizable; so it does not introduce new massless divergences. By contrast, a quadrupole term $\sim (e/m)\partial FW^*\partial W$ gives rise to the singular term $\sim (e/m)\partial FV^*\partial V$, i.e., it introduces new non-renormalizable interactions, some of which do not involve the Stückelberg scalar ϕ .

Instructed by this example, we can lay down a general procedure for studying the dynamics of charged high-spin fields. The same procedure will also apply to other interactions of high-spin fields (gravitational interactions, for instance³). Modulo technical complications due to the presence of auxiliary fields, it is essentially what we followed in the spin 1 case.

A Seven-Step Prescription

Step 1: Write a (non-gauge invariant) massive Lagrangian with minimal number of auxiliary fields (e.g., *à la* Singh and Hagen).

Step 2: Introduce Stückelberg fields and Stückelberg gauge symmetry. Any auxiliary field (appearing in the Lagrangian in step 1) that is not a trace of the high-spin field must be identified as a trace of a Stückelberg field. For such a field one obtains a gauge invariance for free [8].

Step 3: Complexify the fields, if required, and introduce interaction with a new gauge field (e.g., electromagnetism) by replacing ordinary derivatives by their covariant counterparts.

Step 4: Diagonalize all kinetic terms, i.e., get rid of kinetic mixing by field redefinitions and/or by gauge-fixing terms.

Step 5: Look for the most divergent term(s) in the Lagrangian, in an appropriate limit of zero mass and zero coupling. These terms will involve fields that are zero (i.e. gauge) modes of the free kinetic operator before gauge fixing. One needs to take care of the non-commutativity of covariant derivatives and correctly interpret terms proportional to the equations of motion.

Step 6: Try to remove non-renormalizable terms by adding non-minimal terms. This may not always be possible.

Step 7: Find the cutoff of the effective field theory, and interpret the physics implied by the divergent term(s).

The procedure just outlined above also summarizes our paper. Specifically, steps 1 through 5 will be carried over for the case of a charged spin 2 field in Section 2. Section 3 will carry over steps 6 and 7, while the concluding Section 5 will add some more comments on the physics of interacting high-spin fields. Section 4 is somehow at variance with the rest of the paper: it shows that another well-known problem of high-spin fields interacting with electromagnetism, namely the Velo-Zwanziger acausality [9, 10], can also be addressed by our formalism.

³Massive gravity, that is a self-interacting massive spin 2, was studied using a Stückelberg-like formalism in [7].

2 Massive Spin 2 Field Coupled to Electromagnetism

The electromagnetic coupling of a charged massive spin 2 field has been studied in [9, 10, 11, 12, 13]. Such theories are unavoidably fraught with difficulties: Velo and Zwanziger [9, 10], for example, concluded that in a fixed, external electromagnetic background charged massive spin 2 particles show pathological behavior like superluminality and/or acausality. However, as one employs the Stückelberg description, the same underlying physics has a new interpretation. In particular, now the theory can be trusted only up to some intrinsic cutoff, that cannot be sent to infinity. The “pathologies” arise when one (mistakenly) tries to extrapolate an effective description based on a local Lagrangian beyond its regime of validity. In this section and the next we are going to investigate the physics of massive spin 2 field coupled to electromagnetism using this new formalism.

Our starting point is the Pauli-Fierz Lagrangian [14, 15], which is the unique ghost-free, tachyon-free Lagrangian for massive spin 2 field.

$$L = -\frac{1}{2}(\partial_\mu h_{\nu\rho})^2 + (\partial_\mu h^{\mu\nu})^2 + \frac{1}{2}(\partial_\mu h)^2 - \partial_\mu h^{\mu\nu} \partial_\nu h - \frac{m^2}{2}[h_{\mu\nu}^2 - h^2], \quad (8)$$

where $h = h^\mu_\mu$. This Lagrangian does not have any manifest gauge invariance. Now by the field redefinition

$$h_{\mu\nu} \rightarrow \tilde{h}_{\mu\nu} = h_{\mu\nu} + \frac{1}{m} \partial_\mu \left(B_\nu - \frac{1}{2m} \partial_\nu \phi \right) + \frac{1}{m} \partial_\nu \left(B_\mu - \frac{1}{2m} \partial_\mu \phi \right), \quad (9)$$

we create a gauge invariance

$$\delta h_{\mu\nu} = \partial_\mu \lambda_\nu + \partial_\nu \lambda_\mu, \quad (10)$$

$$\delta B_\mu = \partial_\mu \lambda - m \lambda_\mu, \quad (11)$$

$$\delta \phi = 2m \lambda. \quad (12)$$

This gauge invariance, which we will refer to as the Stückelberg symmetry, has been obtained by introducing new (Stückelberg) fields B_μ and ϕ , which can always be gauged away. Yet, as pointed out in the introduction, introducing this redundancy will allow us to unveil the dangerous degrees of freedom and interactions hidden inside the spin 2 action.

It is worth pointing out that one can obtain the Stückelberg version of the Pauli-Fierz Lagrangian by simply starting from a linearized Einstein-Hilbert action in (4+1)D, and then Kaluza-Klein reduce it to (3+1)D [16, 17]. All the fields $h_{\mu\nu}$, B_μ and ϕ thus come from a single higher dimensional massless spin 2 field. The higher dimensional gauge invariance translates itself into the Stückelberg symmetry eqs. (10, 11, 12) in lower dimension. This observation is not trivial; in particular, in the case of higher spins [8], this may help us to construct consistent Lagrangians that can be readily coupled to a $U(1)$ field or gravity, while maintaining at the same time the covariant version of the Stückelberg symmetry.

To couple our massive spin 2 field to electromagnetism we first complexify the Pauli-Fierz Lagrangian with Stückelberg fields, and then replace ordinary derivatives by covariant ones.

$$\partial_\mu \rightarrow D_\mu \equiv \partial_\mu \pm ie A_\mu. \quad (13)$$

We have

$$L = -|D_\mu \tilde{h}_{\nu\rho}|^2 + 2|D_\mu \tilde{h}^{\mu\nu}|^2 + |D_\mu \tilde{h}|^2 - [D_\mu \tilde{h}^{*\mu\nu} D_\nu \tilde{h} + c.c.] - m^2[\tilde{h}_{\mu\nu}^* \tilde{h}^{\mu\nu} - \tilde{h}^* \tilde{h}] - \frac{1}{4} F_{\mu\nu}^2, \quad (14)$$

with

$$\tilde{h}_{\mu\nu} = h_{\mu\nu} + \frac{1}{m} D_\mu \left(B_\nu - \frac{1}{2m} D_\nu \phi \right) + \frac{1}{m} D_\nu \left(B_\mu - \frac{1}{2m} D_\mu \phi \right). \quad (15)$$

Lagrangian (14) now enjoys a covariant Stückelberg symmetry:

$$\delta h_{\mu\nu} = D_\mu \lambda_\nu + D_\nu \lambda_\mu, \quad (16)$$

$$\delta B_\mu = D_\mu \lambda - m \lambda_\mu, \quad (17)$$

$$\delta \phi = 2m \lambda. \quad (18)$$

The above symmetry is obvious only because we had at hand a convenient form of the Stückelberg Lagrangian. This is not so plain in the case of spin 3 and higher, where auxiliary fields, which are not traces of the high-spin field, are unavoidable. At this point it is important to note that the authors in Ref. [18] also considered the gauge invariant description to investigate consistent theories of interactions of massive high spin fields. Starting with a Stückelberg invariant *free* theory, they used minimal substitution: $\partial_\mu \rightarrow D_\mu$, to couple the theory to a gauge field. But as the resulting Lagrangian does *not* have Stückelberg invariance, one has to look for new terms, which must be added to the Lagrangian to recover the invariance. On the other hand, our approach by construction guarantees that Stückelberg symmetry is intact by the minimal substitution. While this may not seem to be a significant achievement for the simple case of spin 2, as one considers higher spins [8] the elegance of our method tremendously facilitates the job of writing down a Stückelberg invariant interacting Lagrangian. In any case, our goal is not just to obtain a gauge invariant description, but to employ the latter to extract a model independent UV cutoff of the effective field theory describing the high spin system, and to show how the well-known pathologies are related to the very existence of a cutoff.

We will now explicitly work out the various terms in the Lagrangian. In doing so we keep in mind that covariant derivatives do not commute.

$$[D_\mu, D_\nu] = \pm ie F_{\mu\nu}. \quad (19)$$

This of course introduces an ambiguity in the definition of the minimal Lagrangian (14). More generally, the Lagrangian is ambiguous because one can always add to it terms vanishing at $F_{\mu\nu} = 0$. We will exploit this ambiguity in Section 3.

After a few integrations by parts we arrive at

$$\begin{aligned} L = & -|\partial_\mu h_{\nu\rho}|^2 + 2|\partial_\mu h^{\mu\nu}|^2 + |\partial_\mu h|^2 - [\partial_\mu h^{*\mu\nu} \partial_\nu h + c.c.] - m^2[h_{\mu\nu}^* h^{\mu\nu} - h^* h] - \frac{1}{4} F_{\mu\nu}^2 \\ & -2(|\partial_\mu B_\nu|^2 - |\partial_\mu B^\mu|^2) + [\phi^* (\partial_\mu \partial_\nu h^{\mu\nu} - \square h) + c.c.] \\ & + 2m[B_\mu^* (\partial_\nu h^{\mu\nu} - \partial^\mu h) + c.c.] + L_{int}. \end{aligned} \quad (20)$$

Here L_{int} is the interaction Lagrangian. It consists of various terms, each one containing at least one power of e , and possibly an inverse power of m ($1/m^4$ at most). These are

the terms we are interested in. Before we write down the interaction terms explicitly, let us concentrate on the kinetic terms.

It is important that the kinetic terms be diagonalized. This makes sure that the propagators in the theory have good high energy behavior⁴. Then we can assign canonical dimensions to the higher-order operators in the interaction Lagrangian, so that we can interpret ours as an effective field theory valid up to some cutoff determined by the most divergent terms in the $m \rightarrow 0$ limit. The kinetic mixings between ϕ and $h_{\mu\nu}, h$ can be eliminated by a standard field redefinition

$$h_{\mu\nu} \rightarrow h_{\mu\nu} - \frac{1}{D-2} \eta_{\mu\nu} \phi, \quad (21)$$

where $D = 4$ is the space-time dimensionality. This also generates a kinetic term for ϕ with the correct sign. The free part of the Lagrangian now looks like

$$\begin{aligned} L_{free} = & L_{PF} - \frac{1}{4} F_{\mu\nu}^2 - 2(|\partial_\mu B_\nu|^2 - |\partial_\mu B^\mu|^2) + 2m \left[B_\mu^* \left(\partial_\nu h^{\mu\nu} - \partial^\mu h + \frac{3}{2} \partial_\mu \phi \right) + c.c. \right] \\ & - \frac{3}{2} |\partial_\mu \phi|^2 - \frac{3}{2} m^2 [\phi^* (h - \phi) + c.c.]. \end{aligned} \quad (22)$$

L_{PF} is the Pauli-Fierz Lagrangian. The mixing terms between B_μ and $h_{\mu\nu}, h, \phi$, which do not look like either a kinetic or a mass mixing can be easily removed by adding first of all a gauge fixing term

$$L_{gf1} = a \left| \partial_\nu h^{\mu\nu} - \frac{1}{2} \partial^\mu h + b B^\mu \right|^2. \quad (23)$$

With the judicious choice $a = -2$ and $b = m$ we get rid of not only the mixing between B_μ and $h^{\mu\nu}$, but also the kinetic mixing $\partial_\mu h^{*\mu\nu} \partial_\nu h$. Notice that for this particular choice of parameters, L_{gf1} does not fix the scalar gauge transformation acting on B_μ, ϕ and given by eqs. (17, 18). This leaves room for adding a second gauge fixing term, which may remove the remaining mixing term, $m \partial^\mu B_\mu^* (h - 3\phi)$. Indeed, this term can be eliminated by a gauge fixing of the form

$$L_{gf2} = c |\partial_\mu B^\mu + d(h - 3\phi)|^2, \quad (24)$$

with $c = -2$ and $d = m/2$. This fully fixes all gauge invariances. Curiously, L_{gf2} also kills the $|\partial_\mu B^\mu|^2$ term, and the mass mixing between h and ϕ . We are finally left with

$$\begin{aligned} L = & h_{\mu\nu}^* (\square - m^2) h^{\mu\nu} - \frac{1}{2} h^* (\square - m^2) h + 2 B_\mu^* (\square - m^2) B^\mu + \frac{3}{2} \phi^* (\square - m^2) \phi \\ & - \frac{1}{4} F_{\mu\nu}^2 + L_{int}. \end{aligned} \quad (25)$$

Here all the kinetic terms are diagonal, so that the propagators, all of which now have the same pole⁵, will behave nicely in the high energy limit. It is worth noting that the

⁴I.e., that all propagators are proportional to $1/p^2$ for momenta $p^2 \gg m^2$.

⁵This is necessary to cancel spurious poles in tree-level physical amplitudes.

“wrong” sign for the kinetic term of h does not necessarily imply a propagating ghost. In fact, such wrong signs usually appear when one performs a covariant gauge fixing.

Now we turn our attention to the interaction terms. Schematically

$$L_{int} = L_8 + L_7 + L_6 + L_5 + L_4, \quad (26)$$

where L_n contains the operators having canonical dimension n , which are multiplied by a factor m^{4-n} . For fixed e , in the high energy limit $m \rightarrow 0$, the higher the n , the more potentially dangerous the operator is. Notice that the gauge fixing terms L_{gf1}, L_{gf2} are regular in the massless limit, so after the minimal substitution $\partial_\mu \rightarrow D_\mu$ they only generate a few harmless, power-counting renormalizable interactions. A look at eq. (15) reveals that by default each ϕ comes with a factor m^{-2} , each B_μ with an m^{-1} , and each $h_{\mu\nu}$ or h with an m^0 . Since the Pauli-Fierz Lagrangian is quadratic in $h_{\mu\nu}$, we can at most have dimension-8 operators, which will necessarily involve a ϕ and a ϕ^* . Next we can have dimension-7 operators containing a ϕ and a B_μ^* , and so on. To pursue our analysis we must explicitly work out these terms, taking good care of appropriate factors and signs. After a tedious but straightforward calculation one finds

$$L_8 = \frac{ie}{m^4} \partial_\rho F^{\mu\rho} D_\mu D_\nu \phi^* D^\nu \phi - \frac{e^2}{4m^4} [5F_{\rho\sigma}^2 \eta_{\mu\nu} + 2F_{\mu\rho} F^\rho{}_\nu] D^\mu \phi^* D^\nu \phi + \frac{e^2}{4m^4} [(\partial^\mu F_{\mu\nu})^2 - 2(\partial_\mu \partial_\rho F^{\mu\nu}) F_\nu{}^\rho] \phi^* \phi - \frac{e^2}{8m^4} (\partial^\rho F_{\mu\nu}^2) [\phi^* D_\rho \phi + \phi D_\rho \phi^*], \quad (27)$$

$$L_7 = - \left\{ \frac{ie}{m^3} D_\mu B_\nu^* [F^{\mu\nu} \square \phi + 2\partial_\rho F^{\mu\rho} D_\nu \phi] + c.c. \right\} + \mathcal{O}(e^2), \quad (28)$$

$$L_6 = - \left\{ \frac{2ie}{m^2} F^{\mu\nu} [\square B_\mu^* B_\nu - 2D_\mu B_\nu^* D^\rho B_\rho + 2D_\mu B_\rho^* D^\rho B_\nu] + c.c. \right\} + \left\{ \frac{ie}{m^2} F^{\mu\nu} [\partial^\rho h_{\mu\rho}^* D_\nu \phi + 2h_{\mu\rho}^* D^\rho D_\nu \phi] + c.c. \right\} + \mathcal{O}(e^2). \quad (29)$$

Let us consider now a scaling limit: $m \rightarrow 0$ and $e \rightarrow 0$, such that $e/m^4 = \text{constant}$. In this limit the Lagrangian simplifies enormously, becoming

$$L = L_{kin} + \left(\frac{e}{m^4} \right) (\partial_\rho F^{\mu\rho}) \left[\frac{i}{2} \partial_\mu \partial_\nu \phi^* \partial^\nu \phi + c.c. \right]. \quad (30)$$

The above equation describes an effective field theory, valid up to a finite cutoff

$$\Lambda_4 = \left(\frac{m^4}{e} \right)^{1/4}. \quad (31)$$

It is the spin-0 Stückelberg (a.k.a. Goldstone) boson that becomes strongly coupled at high energies. This example illustrates the power of the Stückelberg formalism: it focuses precisely on the gauge modes that give rise to the strong coupling. In the unitary gauge these degrees of freedom are obscure, since they manifest as zero modes of the free kinetic operator, and hence strong coupling phenomena cannot be clarified so easily.

Notice that all the terms proportional to e/m^4 and e/m^3 in eqs. (27, 28) are proportional to the equations of motion. Thus one can eliminate them by appropriate *local* field

redefinitions. Of course, one will then introduce terms proportional to e^2/m^8 and e^2/m^6 . But one can hope that different possible terms in the theory somehow conspire to cancel all the $\mathcal{O}(e^2)$ -terms⁶. In this case the only dangerous interactions are those linear in e and *not* proportional to the free equations of motion. If this is so, the Lagrangian reduces to

$$L = L_{kin} + \left\{ \frac{ie}{m^2} F^{\mu\nu} [\partial^\rho h_{\mu\rho}^* \partial_\nu \phi + 2h_{\mu\rho}^* \partial^\rho \partial_\nu \phi + 4\partial_\mu B_\nu^* \partial^\rho B_\rho - 4\partial_\mu B_\rho^* \partial^\rho B_\nu] + c.c. \right\}, \quad (32)$$

so that one obtains a parametrically higher cutoff (in the regime $e \ll 1$, which is the only one where our perturbative procedure makes sense):

$$\Lambda_2 = \frac{m}{\sqrt{e}}. \quad (33)$$

Since the dangerous interaction terms are linear in e , they can be eliminated neither by a perturbative, local field redefinition nor by introducing additional massive degrees of freedom.

3 Adding Non-Minimal Terms

In the most pessimistic scenario the cutoff of our effective field theory is given by $(m^4/e)^{1/4}$. However as we will see now, this cutoff can be pushed to the parametrically higher value of $(m^3/e)^{1/3}$, without the help of extra massive degrees of freedom, by adding appropriate non-minimal terms. The simplest possibility is to add a dipole term.

$$L_{dipole} = ie\alpha F^{\mu\nu} h_{\mu\rho}^* h^\rho{}_\nu + c.c. \quad (34)$$

$$\begin{aligned} \rightarrow & ie\alpha F^\mu{}_\nu \left\{ h_{\mu\rho}^* + \frac{1}{m} (D_\mu B_\rho^* + D_\rho B_\mu^*) - \frac{1}{2m^2} (D_\mu D_\rho + D_\rho D_\mu) \phi^* \right\} \times \\ & \left\{ h^{\rho\nu} + \frac{1}{m} (D^\rho B^\nu + D^\nu B^\rho) - \frac{1}{2m^2} (D^\rho D^\nu + D^\nu D^\rho) \phi \right\} + c.c.. \end{aligned} \quad (35)$$

This non-minimal term will again give rise to operators of various dimensions. In particular the dimension-8 operator reads

$$L_{dipole}^8 = -\frac{2ie\alpha}{m^4} (\partial_\rho F^{\mu\rho}) D_\mu D_\nu \phi^* D^\nu \phi + \frac{e^2\alpha}{m^4} [F_{\rho\sigma}^2 \eta_{\mu\nu} D^\mu \phi^* D^\nu \phi + 2F_{\mu\rho} F^\rho{}_\nu \phi^* D^\mu D^\nu \phi]. \quad (36)$$

It is not surprising that the same operator $\partial_\rho F^{\mu\rho} D_\mu D_\nu \phi^* D^\nu \phi$ shows up in the $\mathcal{O}(e)$ -terms in both eq. (27) and (36). In fact, antisymmetry of $F_{\mu\nu}$ allows only this operator to appear. We can exploit this fact to choose α so that the non-minimal Lagrangian does not contain any terms proportional to e/m^4 . This corresponds to choosing $\alpha = 1/2$.

On the other hand, now we also have dimension-7 operators

$$L_{dipole}^7 = - \left\{ \frac{2ie\alpha}{m^3} F^{\mu\nu} (D_\mu B_\rho^* + D_\rho B_\mu^*) D^\rho D_\nu \phi + c.c. \right\} + \mathcal{O}(e^2). \quad (37)$$

⁶This may happen in particular if other interactions exist, that are linear in $F_{\mu\nu}$ and mix the spin 2 field with other more massive degrees of freedom. By integrating out these additional degrees of freedom, one ends up with additional EM interactions at $\mathcal{O}(e^2)$, that involve only the spin 2 field.

It is easy to see that no choice of α can cancel all dimension-7 terms in the minimal Lagrangian; not even to $\mathcal{O}(e)$. To make it even worse, the non-minimal terms introduce new dimension-7 operators.

Thus the best we can do by adding the dipole term is to eliminate terms proportional to e/m^4 . In such a case, we take the $m \rightarrow 0$, $e \rightarrow 0$ limit, keeping $e/m^3 = \text{constant}$. The non-minimal Lagrangian thereby reduces to

$$L = L_{kin} - \left\{ \frac{ie}{m^3} [(\partial_\rho F^{\mu\rho})(\partial_\mu B_\nu^* \partial^\nu \phi + B_\mu^* \square \phi) + (\partial_\rho F^{\mu\nu}) B_\mu^* \partial_\rho \partial_\nu \phi] + c.c. \right\}. \quad (38)$$

Now both the longitudinal modes ϕ and B_μ participate in the strong coupling dynamics. But the cutoff is parametrically higher than that of the minimal theory:

$$\Lambda_3 = \left(\frac{m^3}{e} \right)^{1/3} \gg \Lambda_4. \quad (39)$$

One may be tempted by the success of the above procedure to add other non-minimal terms to further raise the cutoff scale. But that does not help much, since all other possible non-minimal terms contain at least dimension-6 operators to begin with (6 is a quadrupole term). After the Stückelberg procedure, addition of, say, a quadrupole term produces operators up to dimension 10. Although we can cancel the dimension-10 operators by a clever linear combination of the possible quadrupole terms, we cannot get rid of the dimension-9 operators. Thus not only that we gain nothing, but actually we lower the UV cutoff of the theory. The conclusion, therefore, is that Λ_3 is the highest we can raise the cutoff to without adding additional massive degrees of freedom. Notice that this cutoff is still lower than the “optimistic” one, $\Lambda_2 = m/\sqrt{e}$.

4 Superluminality and Absence Thereof

Many years ago Velo and Zwanziger [9, 10] discovered that charged, massive fields of spin higher than one exhibit pathological behavior in external (constant) electromagnetic fields. The high-spin field may have modes that propagate faster than light, or the number of propagating degrees of freedom may be different than that of the free theory, or the Cauchy problem may become ill-posed. All these pathologies are due to the fact that the free kinetic term of high-spin fields exhibit gauge invariances, so that it has zero modes. In the presence of electromagnetic interactions, these modes acquire a non-vanishing but non-canonical kinetic term, which may allow for some of them to propagate superluminally, or which may not even be hyperbolic. The formalism we employed in this paper is tailored to single out the dynamics of precisely these zero modes, which are none other than the Stückelberg fields. So, our formalism should allow us to recover the results of Velo and Zwanziger, generalize them, and simplify their derivation.

A first simplification is achieved by taking a convenient scaling limit:

$$e \rightarrow 0, \quad m \rightarrow 0, \quad \frac{e F_{\mu\nu}}{m^2} = \text{constant}. \quad (40)$$

We further notice that in a constant external electromagnetic background the interaction terms L_7 and L_6 [eqs. (28, 29)] either vanish or become proportional to the free equations

of motion of the B_μ field. We can thus limit our analysis to solutions where B_μ propagates on the light cone, so that only the scalar ϕ exhibits non-standard dynamics. By doing so, we may miss some non-standard solutions in which the vector field B_μ propagates outside the light cone. So the following analysis will be able to exhibit acausality and other defects of the spin 2 system, but not to exclude them completely (they may disappear for the scalar sector but reappear in the vector-scalar system).

Keeping this caveat in mind, we notice first that after taking the limit (40) and setting the B_μ field on shell, the only relevant interaction terms in our non-minimal Lagrangian, for a generic dipole coefficient α , are

$$L_\phi = -\frac{e^2}{4m^4}[(5 - 4\alpha)F_{\rho\sigma}F^{\rho\sigma}\eta^{\mu\nu} + (2 + 8\alpha)F^{\mu\rho}F_\rho{}^\nu]\partial_\mu\phi^*\partial_\nu\phi. \quad (41)$$

These terms carry one derivative of ϕ , and another of ϕ^* , so that in a constant electromagnetic background they behave like additional kinetic terms for the field ϕ . This can potentially give rise to superluminal propagation. As long as its kinetic term is concerned, ϕ will experience a new effective background metric, different from Minkowski:

$$\tilde{\eta}^{\mu\nu} = \left(\frac{3}{2} + \beta F_{\rho\sigma}F^{\rho\sigma}\right)\eta^{\mu\nu} + \gamma F^{\mu\rho}F_\rho{}^\nu, \quad (42)$$

where we have defined

$$\beta \equiv \frac{e^2}{4m^4}(5 - 4\alpha), \quad \gamma \equiv \frac{e^2}{4m^4}(2 + 8\alpha). \quad (43)$$

Notice that eq. (42) gives the *contravariant* metric. For $\gamma = 0$ the background metric $\tilde{\eta}^{\mu\nu}$ is proportional to the Minkowski metric, therefore ϕ does not propagate superluminally. This case corresponds to a non-minimal Lagrangian with $\alpha = -1/4$. Notice also that even for this value of α the system ceases to be hyperbolic in a strong field; precisely, when $e^2 F_{\mu\nu}F^{\mu\nu}/m^4 = -1$.

However, things can be very different for other values of α , for example when $\alpha = 0$, that corresponds to the minimal Lagrangian. Let us consider two special cases of constant electromagnetic background: a constant magnetic field, and a constant electric field.

Constant Magnetic Field: $\vec{B} = \hat{k}B$

In this case the background metric (42) reduces to

$$2\tilde{\eta}^{\mu\nu} = \text{diag}[-3 - 4\beta B^2, 3 + (4\beta - 2\gamma)B^2, 3 + (4\beta - 2\gamma)B^2, 3 + 4\beta B^2]. \quad (44)$$

On a plane perpendicular to \vec{B} , the wave speed is different from c in general; it is given by

$$\frac{v^2}{c^2} = 1 - \frac{2\gamma B^2}{4\beta B^2 + 3} = 1 - \frac{e^2(1 + 4\alpha)B^2}{e^2(5 - 4\alpha)B^2 + 3m^4}. \quad (45)$$

For different values of the parameter α one finds the following results

<u>Value of α</u>	<u>Result</u>
$\alpha \geq 5/4$	Superluminal propagation for $B^2 > \frac{3m^4/e^2}{4\alpha-5}$.
$1/2 < \alpha < 5/4$	Loss of hyperbolicity for finite B .
$-1/4 < \alpha \leq 1/2$	Subluminal propagation for any B .
$\alpha = -1/4$	$v = c$.
$\alpha < -1/4$	Superluminal propagation for any B .

We see that the choice $\alpha < -1/4$ is downright pathological, since it gives superluminal propagation even for infinitesimally small B . On the other hand, other values of α are good, at least for sufficiently small values of B . In particular, for $\alpha = -1/4$ the wave speed is c , as expected. The superluminality reported by Velo and Zwanziger [9, 10] corresponds to the case $\alpha \geq 5/4$. In fact, one obtains the same threshold value of $2m^2/3e$ by setting $\alpha - 3 = -1/16$.

At this point one may argue that both our minimal Lagrangian ($\alpha = 0$), and the improved one with higher cutoff ($\alpha = 1/2$) are free of pathologies, because subluminal propagation by itself is harmless. But before we draw a conclusion we need to consider other cases.

Constant Electric Field: $\vec{E} = \hat{k}E$

Here the background metric (42) reduces to

$$2\tilde{\eta}^{\mu\nu} = \text{diag}[-3 + (4\beta - 2\gamma)E^2, 3 - 4\beta E^2, 3 - 4\beta E^2, 3 - (4\beta - 2\gamma)E^2]. \quad (46)$$

Again, the wave speed on a plane perpendicular to \vec{E} may not be c . Namely:

$$\frac{v^2}{c^2} = 1 + \frac{2\gamma E^2}{(4\beta - 2\gamma)E^2 - 3} = 1 + \frac{e^2(1 + 4\alpha)E^2}{e^2(4 - 8\alpha)E^2 - 3m^4}. \quad (47)$$

Now we have the following results:

<u>Value of α</u>	<u>Result</u>
$\alpha \geq 5/4$	Subluminal propagation for any E .
$\alpha < 5/4$	Loss of hyperbolicity for finite E .
$-1/4 < \alpha < 1/2$	Superluminal propagation for $E^2 \geq \frac{3m^4/e^2}{4-8\alpha}$.
$\alpha = -1/4$	$v = c$.
$\alpha < -1/4$	Superluminal propagation for $0 < E^2 < \frac{3m^4/e^2}{4-8\alpha}$.

In this case too the domain $\alpha \geq -1/4$ is safe, at least for sufficiently weak external fields. This is probably all we should ask from our effective theory. To trust it for external backgrounds $eF/m^2 \sim 1$ would require us to make the unreasonably strong assumption that we can safely neglect all nonlinear, higher order corrections in the external electromagnetic field. It is worth noticing that even non-minimally coupled massive spin 3/2 fields always manifest inconsistencies for large external fields [19], although they can be safe for small fields; therefore, they too can make sense at most as effective low-energy descriptions of some more fundamental theories. A particular instance of non-minimally coupled charged spin 1 field theory, arising from open strings, was studied in [20]. The same phenomenon arises there as well: strong fields are pathological,⁷ but weak fields are not.

Generic Constant EM Field

For a generic *constant* electromagnetic background the same conclusions hold. Indeed, whenever $\vec{E} \cdot \vec{B} = 0$ and $|\vec{E}| \neq |\vec{B}|$, there exists a frame in which the field is either purely electric or purely magnetic, so that our previous analysis applies trivially.

When $\vec{E} \cdot \vec{B} \neq 0$, there exists a frame in which \vec{E} is parallel to \vec{B} . Then, an analysis similar to our previous cases tells us that in the range $\alpha \in [-1/4, 1/2]$ superluminal propagation exists only in strong external fields; namely:

$$\vec{E}^2 - \vec{B}^2 \geq \frac{3}{4\beta} + \frac{\gamma}{2\beta} \vec{E}^2. \quad (48)$$

This bound is weakest at $\vec{B} = 0$.

Finally, when the fields are perpendicular and equal in norm, $\vec{E} \cdot \vec{B} = 0$, $|\vec{E}| = |\vec{B}| \equiv E$, one can choose a frame where the contravariant metric (42) becomes two-by-two block-diagonal

$$\tilde{\eta} \sim \begin{pmatrix} A & 0 \\ 0 & I \end{pmatrix}, \quad (49)$$

$$A = \begin{pmatrix} -1 - \frac{2}{3}\gamma E^2 & -\frac{2}{3}\gamma E^2 \\ -\frac{2}{3}\gamma E^2 & 1 - \frac{2}{3}\gamma E^2 \end{pmatrix}. \quad (50)$$

In this metric background signals propagate with three characteristic speeds: $c/\sqrt{1 + \frac{2}{3}\gamma E^2}$ or $c(\frac{2}{3}\gamma E^2 \pm 1)/(1 + \frac{2}{3}\gamma E^2)$. Thus superluminality only appears for $\gamma < 0$, i.e., $\alpha < -1/4$.

5 Conclusion

This paper illustrates the power of the Stückelberg method for understanding the dynamics of interacting high-spin fields. The method renders free massive theories invariant under the same gauge symmetries of massless theories by introducing redundant degrees of freedom. They can be eliminated by using the gauge invariance, thereby recovering the

⁷And unreliable, since open strings in external electric fields are unstable due to Schwinger pair production at $|E| \sim M_{string}^2/e \leq m^2/e$ [21].

usual [2] free Lagrangians. The Stückelberg fields also make the *interacting* theory gauge invariant. By appropriate covariant gauge fixings, field redefinitions and scaling limits, one is then able to extract from the full theory the sub-sector responsible for all pathologies of the theory: strong coupling at finite energy scale, acausal propagation in external fields etc. The Stückelberg formalism unifies the description of all these phenomena.

We illustrated our point by analyzing only one of the simplest high-spin systems: a spin 2 field coupled to electromagnetism. We found out that the system possesses an intrinsic UV cutoff, no higher than $\Lambda_2 = m/\sqrt{e}$, and that its scalar Goldstone/Stückelberg sector is where the acausal behavior found by Velo and Zwanziger manifests itself. Generalizations of this example are under way [8]. They are both intriguing and subtle, since starting from spin 3, the Stückelberg sector itself contains spin 2 fields which, in particular, does not admit a smooth massless limit [22]. What happens then when we try to take a scaling limit where $m \rightarrow 0$? Irrespective of the answer to this question, the behavior of high-spin fermions should not differ substantially from that of bosons, since the source of strong-coupling pathologies is the existence of gauge invariances in the massless, free kinetic term, independently of their statistics (see e.g. refs [5, 6, 9, 10, 19]).

Gravitational interactions of high-spin fields are also of paramount importance, since they are truly universal. If an intrinsic cutoff is found for such interactions, it signals the ultimate limit of any local effective field theory description of interacting high-spin massive fields.

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